

**WORCESTER POLYTECHNIC INSTITUTE
TWENTY-NINTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 18, 2016
TEAM EXAM QUESTION SHEET**

DIRECTIONS: Please write your answers on the **TEAM ANSWER SHEET** provided. This part of the contest is 45 minutes. All 14 problems are counted equally. Calculators and other electronics **MAY NOT** be used.

1. Find pairs $(\sin(t), \cos(t))$ such that $\tan(2t) = 120/119$.

2. If $A = \begin{pmatrix} 3\sqrt{3}/2 & -3/2 \\ 3/2 & 3\sqrt{3}/2 \end{pmatrix}$, find A^6 .

3. If P is a matrix $\begin{pmatrix} 2/3 & 1/4 \\ 1/3 & 3/4 \end{pmatrix}$ and \vec{x} is a column vector such that a) its entries add up to 1, and b) $P\vec{x} = \vec{x}$, then what is \vec{x} ?

4. Ten turns of a wire are helically wrapped about a cylinder with outer circumference 4 inches and length 9 inches. The ends of the wire coincide with the ends of the same cylindrical element. Find the length of the wire.

5. If the 3 cube roots of a real number are r , $a + bi$ and $a - bi$, express a and b in terms of r .

6. Consider a *cone* formed by connecting all points on the conic section in the x - y plane

$$25x^2 - 100x + 9y^2 - 72y + 19 = 0$$

with the point $(-1, 4, 10)$ (often called its *apex*). Determine its volume.

7. If $f(x)$ is a cubic polynomial function and it is known that
 a) $2-x$, $x-3$ and $x+4$ divide it b) $f(1) = -20$ what is $f(x)$?
8. If $n > 2$, consider the $(n \times n)$ -matrix whose entries, in order, row by row, are $1, 2, 3, 4, \dots, n^2$
 What is its determinant?
9. A cylindrical pipe is laying horizontally on its side. It has an inner diameter of 12 feet and a length of 50 feet. It has water in it to a depth of 9 feet. What is the volume of water in the pipe?
10. How many integer solutions (x, y) are there to the equation $x^2 - 3y^2 = 17$?
11. For x real, if $x^a x^b = 1$ and x is not -1 or $+1$, evaluate $4a - b^2 + a^2 + 4b - 10$
12. In a circle of radius 1, three circles of equal radius are inscribed such that they are of maximal size, but do not overlap (but might touch). Another circle, concentric to the first one, is inscribed in a way that it is of maximal size and does not overlap with (but might touch) any of the three other inscribed circles. What is the radius of this circle?
13. Find all numbers x, y, z such that the sum of the product of any two added to the third one equals 2.
14. Please find all θ in $[0, 2\pi)$ which satisfy

$$\cos^2(\theta) + 2.5\sin(\theta) = 2.$$