

WORCESTER POLYTECHNIC INSTITUTE
THIRTIETH ANNUAL INVITATIONAL MATH MEET
OCTOBER 17, 2017
TEAM EXAM QUESTION SHEET WITH ANSWERS

DIRECTIONS: Please write your answers on the **TEAM ANSWER SHEET** provided. This part of the contest is 45 minutes. All 14 problems are counted equally. Calculators and other electronics **MAY NOT** be used.

1. Simplify to a single number:

$$\log_2(3) \log_3(4) \log_4(5) \log_5(6) \log_6(7) \log_7(8)$$

Ans: 3

2. Calculate $\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots$

Ans: 2

3. The parallel lines connect a vertex to a midpoint. What fraction of the total area is the shaded area?

Ans: 1/3

4. I have 9 coins in my pocket: 3 nickels, 2 half dollars, 2 quarters, 2 pennies.
If I randomly take 4 coins out what is the probability that I have 2 nickels?

Ans: 5/14

5. List all numbers in Z_{15} (integers mod 15) which have multiplicative inverses.

Ans: 1,2,4,7,8,11,13,14

6. Find the smallest positive value x , in degrees, such that
 $\cos^2 x + \sin^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \csc^2 x = 31$.

Ans: 15°

7. If the three cube roots of 1 are 1, w , and w^2 determine the value of
 $(1 + w - w^2)^3 + (1 - w + w^2)^3$ as a single real number.

Ans: -16

8. An isosceles right triangle is removed from each corner of a square piece of paper so that a rectangle remains. What is the length of a diagonal of the rectangle if the areas of all the cut off pieces is a total of 200?

Ans: 20

9. Simplify $(-1 - \sqrt{3}i)^9$ Ans: 512
10. For what x is it true that $\sqrt{x + 2\sqrt{x-1}} + \sqrt{x - 2\sqrt{x-1}} = 2\sqrt{x-1}$ Ans: all $x \geq 1$
11. Simplify to a single number: $\sqrt[3]{5 + 2\sqrt{13}} + \sqrt[3]{5 - 2\sqrt{13}}$ Ans: 1
12. Find all ordered pairs of real numbers (x,y) such that
 $5^{y-x}(x+y) = 1$ and $(x+y)^{x-y} = 5$ Ans: (3,2) and (-2/5, 3/5)
13. Suppose we have a cone formed in the following way: all points on the ellipse in the x - y plane whose equation is $\frac{x^2}{25} + \frac{y^2}{81} = 1$ are connected by line segments to the point $(-5,9,7)$
What is the volume of this cone? Ans: 105π
14. What is the shaded area below? Ans: $2/3$