

**WORCESTER POLYTECHNIC INSTITUTE
THIRTY-SECOND ANNUAL INVITATIONAL MATH MEET
OCTOBER 15, 2019
TEAM EXAM QUESTION SHEET WITH ANSWERS**

DIRECTIONS: Please write your answers on the **TEAM ANSWER SHEET** provided. This part of the contest is 45 minutes. All 14 problems are counted equally. Calculators and other electronics **MAY NOT** be used.

1. A polynomial has a remainder of 8 when divided by $x-2$, remainder of 3 when divided by $x-3$, and a remainder of -6 when divided by $x-4$. What remainder will it have when divided by $(x-2)(x-3)(x-4)$? **ans:** $-2x^2 + 5x + 6$

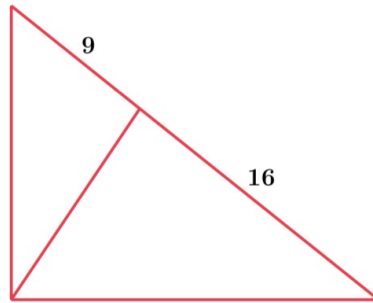
2. What is the 1603rd digit in the decimal for $1/17$? **ans:** 8

3. Evaluate
$$\frac{1+3+5+\dots+739}{741+743+\dots+1479}$$
ans: $1/3$

4. Twenty integers are in a geometric progression. The first five are 7-digit integers, the next six are 8-digit integers, the next five are 9-digit integers and the last four have ten digits each. What is the smallest integer in the sequence? (You do not need to write down the common ratio between consecutive terms.)
ans: 1572864.

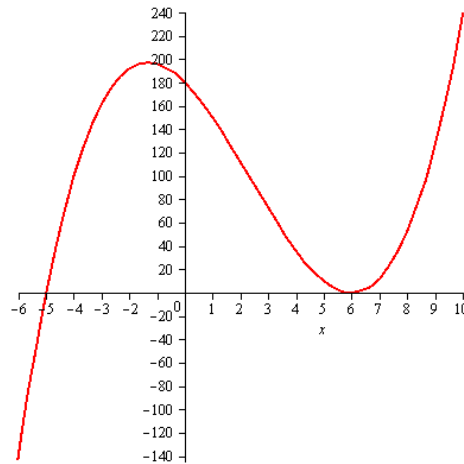
5. In order, Ali, Bobby and Carmen take turns flipping the same fair coin repeatedly. The first one to toss a head wins. What is the probability that Ali wins? **ans:** $4/7$

6. Find the area of the largest triangle shown:



ans: 150

7. Find a cubic polynomial which fits the graph shown. Your answer must be in standard form $y = Ax^3 + Bx^2 + Cx + D$



ans: $y = x^3 - 7x^2 - 24x + 180$

8. A projectile follows a path (x, y) for $t \geq 0$ of

$$\begin{array}{ll} y = -8t^3 + 120t^2 + 800t & \text{height in feet above the ground, and} \\ x = 2t & \text{horizontal distance in feet} \end{array}$$

How far, horizontally, will it go before striking the ground?

ans: 40 feet

9. The temperature T of an object at time $t \geq 0$ is given by $T(t) = 20e^{-4t} + 10$. When will the object have cooled to half of its initial temperature? **ans:** $t = \ln(4)/4 = \ln(2)/2$

10. Consider a right circular cone with equation $z^2 = (x^2 + y^2)/3$ and $0 \leq z \leq 10$. What is its surface area? **ans:** $100\pi(3 + 2\sqrt{2})$

11. What is the smallest positive integer X such that all the digits in the product $9X$ are 1's? **ans:** 12345679

12. Consider the region in the first quadrant of the x - y plane whose Cartesian description is

$$0 \leq x \leq \frac{5}{\sqrt{3}} \quad \text{and} \quad \frac{x}{\sqrt{3}} \leq y \leq 5 \quad \text{Describe it instead in Polar Coordinates}$$

ans: $\pi/3 \leq \theta \leq \pi/2$ and $0 \leq r \leq 5/\sin(\theta)$

13. if $i = \sqrt{-1}$ what is the value of $i^{2019} + i^{2020} + i^{2021} + i^{2022}$? **ans:** 0

14. A standard right circular cone, Cartesian equation $z^2 = x^2 + y^2$ has a ball of radius 5 placed into it. What will be the height above the x - y plane of the center of the ball? **ans:** $2\sqrt{5}$