Exercise 1:
Let $(X, \rho)$ be a metric space and $S$ and $T$ two non-empty subsets of $X$. Define
\[
d(S, T) = \max \{ \sup_{x \in S} \inf_{y \in T} \rho(x, y), \sup_{y \in T} \inf_{x \in S} \rho(x, y) \}.
\]
Show that $d(S, T) = 0$ if and only if $S$ and $T$ have the same closure.

Exercise 2:
Show that for every set $S \subset \mathbb{R}$ there exists a Borel set $B$ such that $S \subset B$ and $m^*(S) = m^*(B)$, where $m^*$ is the Lebesgue outer measure. Then show that for such $S$ and $B$ with $m^*(S) < \infty$, $S$ is measurable if and only if $m^*(B \setminus S) = 0$.

Exercise 3:
Suppose $f_n, g_n$ are Lebesgue measurable functions on $\mathbb{R}$, with $f_n, g_n \geq 0 \ \forall n \in \mathbb{N}$. Suppose also that $f_n \to f$ a.e., $g_n \to g$ a.e.,
\[
\int f_n \to \int f < \infty,
\]
and
\[
\int g_n \to \int g < \infty.
\]
Prove or give a counterexample: if $\{f_n g_n\}$ is bounded in $L^1$, then
\[
\int f_n g_n \to \int fg.
\]