Robotic manipulators in interventional medicine and surgery

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Postdoc in Mechanical Engineering, Vanderbilt University - 2017
PhD in Robotics, Cognition and Interaction Technologies, IIT - 2015
BS/MS in Computer Engineering, University of Catania – 2011
Liceo Scientifico Stanislao Cannizzaro, Vittoria RG
Class of 2005

My high school math teacher
Robotic technology today

da Vinci surgical System
(© Intuitive Surgical, Inc.)
The first surgical robot
Robot kinematics

Goal: describe the *pose* of the robot (position and orientation)

Fig. 1: PUMA 560 in the zero position with attached coordinates frames shown\textsuperscript{17}

https://www.youtube.com/watch?v=tjOhGqOHfhg
Robot kinematics

**Goal:** describe the *pose* of the robot (position and orientation)

Transformation matrix between two consecutive joints:

\[ T_{i+1}^i = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

Transformation between robot base and end effector:

\[ T_0^6 = T_0^1 T_1^2 T_2^3 T_3^4 T_4^5 T_5^6 \]

Fig. 1: PUMA 560 in the zero position with attached coordinates frames shown\(^1\)
It may get complex very quickly...

\[
0^T \mathbf{r} = \begin{bmatrix}
0 & 0 & 0 & p_x \\
0 & 0 & 0 & p_y \\
0 & 0 & 0 & p_z \\
1 & 1 & 1 & 1
\end{bmatrix}
\]

\[
\begin{align*}
0^T r_1 &= c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6), \\
0^T r_2 &= s_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_6] - c_1(s_4c_5c_6 + c_4s_6), \\
0^T r_3 &= -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6, \\
0^T r_4 &= s_1[c_{23}(-c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6), \\
0^T r_5 &= s_1[c_{23}(-c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6), \\
0^T r_6 &= -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6, \\
0^T r_7 &= -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5, \\
0^T r_8 &= -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5, \\
0^T r_9 &= s_{23}c_4s_5 - c_{23}c_5, \\
0^T r_{10} &= c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1, \\
0^T r_{11} &= s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1, \\
0^T r_{12} &= -a_3s_{23} - a_2s_2 - d_4c_{23}, \\
\end{align*}
\]

Fig. 1: PUMA 560 in the zero position with attached coordinates frames shown\(^7\)
Inverse Kinematics

**Goal:** calculate the joint angles for a desired pose

\[
\begin{align*}
    r_{11} &= c_1 [c_{23}(c_4c_5c_6 - s_4s_5) - s_23s_5c_5] + s_1(s_4c_5c_6 + c_4s_6), \\
    r_{21} &= s_1 [c_{23}(c_4c_5c_6 - s_4s_6) - s_23s_5c_6 - c_1(s_4c_5c_6 + c_4s_6), \\
    r_{31} &= -s_23(c_4c_5c_6 - s_4s_6) - c_23s_5c_6, \\
    r_{12} &= c_1 [c_{23}(-c_4c_5s_6 - s_4c_6) + s_23s_5s_6] + s_1(c_4c_6 - s_4c_5s_6), \\
    r_{22} &= s_1 [c_{23}(-c_4c_5s_6 - s_4c_6) + s_23s_5s_6] - c_1(c_4c_6 - s_4c_5s_6), \\
    r_{32} &= -s_23(-c_4c_5s_6 - s_4c_6) + c_23s_5s_6, \\
    r_{13} &= -c_1(c_23c_4s_5 + s_23c_5) - s_1s_4s_5, \\
    r_{23} &= -s_1(c_23c_4s_5 + s_23c_5) + c_1s_4s_5, \\
    r_{33} &= s_23c_4s_5 - c_23c_5, \\
    p_x &= c_1 [a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1, \\
    p_y &= s_1 [a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1, \\
    p_z &= -a_3s_{23} - a_2s_2 - d_4c_{23}.
\end{align*}
\]
A geometric example

\[ \mathbf{W} = (p_{Wx}, p_{Wy}) \]

\[ \theta_1 = ? \quad \theta_2 = ? \]

Law of Cosines

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

\[ \cos A = \frac{b^2 + c^2 - a^2}{2bc} \]

\[ \cos(\pi - \theta_2) = \frac{a_1^2 + a_2^2 - (p_{Wx}^2 + p_{Wy}^2)}{2a_1a_2} \]

\[ \cos(\theta_2) = -\frac{a_1^2 + a_2^2 - (p_{Wx}^2 + p_{Wy}^2)}{2a_1a_2} \]
A geometric example

\[ \vec{W} = (p_{Wx}, p_{Wy}) \]

\[ \theta_1 = \alpha \pm \beta \]

\[ \alpha = \tan^{-1} \left( \frac{p_{Wy}}{p_{Wx}} \right) \]

\[ \beta = \cos^{-1} \left( \frac{a_1^2 + p_x^2 + p_y^2 - a_2^2}{2a_1 \sqrt{p_{Wx}^2 + p_{Wy}^2}} \right) \]

\[ \cos(\theta_2) = -\frac{a_1^2 + a_2^2 - (p_{Wx}^2 + p_{Wy}^2)}{2a_1 a_2} \]
Inverse Kinematics

Even if we find an analytic inverse, the problem may still have zero/multiple/infinite solutions!

\[
\begin{align*}
    r_{11} &= c_1 [c_{23} (c_4 c_5 c_6 - s_4 s_5) - s_{23} s_5 c_5] + s_1 (s_4 c_5 c_6 + c_4 s_5), \\
    r_{21} &= s_1 [c_{23} (c_4 c_5 c_6 - s_4 s_5) - s_{23} s_5 c_6 - c_1 (s_4 c_5 c_6 + c_4 s_5)], \\
    r_{31} &= -s_{23} (c_4 c_5 c_6 - s_4 s_6) - c_{23} s_5 c_6, \\
    r_{12} &= c_1 [c_{23} (-c_4 c_5 s_6 - s_4 c_6) + s_{23} s_5 s_6] + s_1 (c_4 c_6 - s_4 c_5 s_6), \\
    r_{22} &= s_1 [c_{23} (-c_4 c_5 s_6 - s_4 c_6) + s_{23} s_5 s_6] - c_1 (c_4 c_6 - s_4 c_5 s_6), \\
    r_{32} &= -s_{23} (-c_4 c_5 s_6 - s_4 c_6) + c_{23} s_5 s_6, \\
    r_{13} &= -c_1 (c_{23} c_4 s_5 + s_{23} c_5) - s_1 s_4 s_5, \\
    r_{23} &= -s_1 (c_{23} c_4 s_5 + s_{23} c_5) + c_1 s_4 s_5, \\
    r_{33} &= s_{23} c_4 s_5 - c_{23} c_5,
\end{align*}
\]

\[
\begin{align*}
    p_x &= c_1 [a_2 c_2 + a_3 c_{23} - d_4 s_{23}] - d_3 s_1, \\
    p_y &= s_1 [a_2 c_2 + a_3 c_{23} - d_4 s_{23}] + d_3 c_1, \\
    p_z &= -a_3 s_{23} - a_2 s_2 - d_4 c_{23}.
\end{align*}
\]

https://www.youtube.com/watch?v=0nZ7vTxpLQU
Endoscope kinematics

Endoscope kinematics


\[
T_{j+1}^j = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \cos(\kappa s) & -\sin(\kappa s) & (\cos(\kappa s) - 1)/\kappa \\
0 & \sin(\kappa s) & \cos(\kappa s) & \sin(\kappa s)/\kappa \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
Continuum robots in surgery

Gilbert 2016

Swaney 2017
The Rise of Robots in the Operating Room | Dr. Robert Webster III | TEDxNashvilleSalon

https://www.youtube.com/watch?v=Mr4xEH11N5A
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