

**WORCESTER POLYTECHNIC INSTITUTE
TWENTY-SEVENTH INVITATIONAL MATH MEET
OCTOBER 21, 2014
INDIVIDUAL EXAM QUESTION SHEET**

DIRECTIONS: Please write your answers on the **Individual Answer Sheet** provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators **MAY NOT** be used.

1) Simplify

$$\left(\frac{(1 * 2 * 4 + 2 * 4 * 8 + 3 * 6 * 12 + \dots)}{(1 * 3 * 9 + 2 * 6 * 18 + 3 * 9 * 27 + \dots)} \right)^{1/3}$$

2) For what integer **a** does $x^2 - x + a$ divide into $x^{13} + x + 90$?

3) What is the binary form of the 6th Fibonacci number?

4) Find two numbers whose difference and quotients are each equal to 5

5) How many different five digit numbers can be constructed using the digits {1,1,1,4,7}?

6) Simplify $1 + 2x + 3x^2 + 4x^3 + \dots$ if $|x| < 1$

7) What is the sum of $102+104+ . . . +196+198+200+202+204$?

8) Simplify $10^{-\log_{100} 16}$

9) Find a value of k such that the following system is consistent

$$x + y = 1$$

$$kx + y = 2$$

$$x + ky = 3$$

10) During a period of days, it was observed that when it rained in the afternoon, it had been clear in the morning, and when it rained in the morning it was clear in the afternoon. It rained on 9 days, and was clear on 6 afternoons and 7 mornings. How long was this period, in days?

11) For what real values of x does the following equation hold?

$$\sqrt{x + 2\sqrt{x-1}} + 4\sqrt{x - 2\sqrt{x-1}} = 2\sqrt{x-1} + 15$$