

WORCESTER POLYTECHNIC INSTITUTE
TWENTY-NINTH INVITATIONAL MATH MEET
OCTOBER 18, 2016
INDIVIDUAL EXAM QUESTION SHEET WITH ANSWERS

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes long. Questions 1-4 are each worth 1 point. Questions 5-8 are each worth 2 points. Questions 9-11 are each worth 3 points. Calculators and other electronics **MAY NOT** be used.

1 Point Each:

1. Find an equation for the circle which passes thru the points **(1, -6), (4, -15), and (9, -10)**

Ans: $(x - 4)^2 + (y + 10)^2 = 25$ or $x^2 - 8x + y^2 + 20y + 91 = 0$

2. Simplify to standard form: $\frac{27 + 8i}{3 + 2i^3}$

Ans: $5 + 6i$

3. What is the coefficient of a^4bc^2 in the expansion of $(a + 2b + c)^7$?

Ans: 210

4. Consider the graph whose equation is $4x^2 + 9y^2 - 16x + 18y - 11 = 0$

What is the sum of the distances from (x, y) on this graph to $(2 - \sqrt{5}, -1)$ and $(2 + \sqrt{5}, -1)$?

Ans: 6

2 Points Each:

5. Computer passwords are to consist of a string of 6 symbols taken from the digits **0, 1, 2, ...9** and the lower case letters **a, b, ...z**. How many computer passwords have a repeated symbol?

Ans: 774372096

6. If $i = \sqrt{-1}$ what is $(1+i)^{16}$?

Ans: 256

7. Assume that fresh watermelons are 99% water. One ton of fresh watermelons are shipped. Some dehydration occurs during transit. When the watermelons arrive they are 98% water. How much do they now weigh?

Ans: $\frac{1}{2}$ ton

8. You are looking for the only 5 digit number which, when multiplied by 9, results in the digits in reverse order. What is that number?

Ans: 10989

3 Points Each:

9. If $f(x) = \sqrt{x+2\sqrt{x-1}} - \sqrt{x-2\sqrt{x-1}}$ $x \geq 1$ find $\sum_{n=1}^{100} f(n)$

Ans: 198

10. Consider the geometric series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$. How many terms are needed so that the partial sum is within 0.001 of the sum of the series?

Ans: 7

11. What is $10510100501^{1/5}$?

Ans: 101