WPI Mathematical Sciences Grants and Awards 2014 -1015

Higher-Order Methods for Interface Problems with Non-Aligned Meshes

National Science Foundation

Sarkis, M.

2015-2018

Interface problems arise in several applications including heart models, cochlea models, aquatic animal locomotion, blood cell motion, front-tracking in porous media flows and material science, to name a few. One of the difficulties in these problems is that solutions are normally not smooth across interfaces, and therefore standard numerical methods will lose accuracy near the interface unless the meshes align to it. However, it is advantageous to have meshes that do not align with the interface, especially for time dependent problems where the interface moves with time. Re-meshing at every time step can be prohibitively costly, can destroy the structure of the grid, can deteriorate the well-conditioning of the stiffness matrix, and affect the stability of the problem. The first problem studied will involve new stable and higher-order accurate Finite Element - Immersed Boundary Methods (FE-IBM) for evolution problems where the interface moves with time. The second problem studied is the design and analysis of robust higher-order discretizations for interface problems with high-contrast discontinuous diffusion coefficients. Benefits of the project include the strengthening of connections between numerical analysis and other areas of science and engineering, particularly bioengineering, porous media flows, material sciences and parallel computing. This project will impact the development of numerical algorithms used in the fluid-structure interaction communities. A broader impact will be the training of graduate and undergraduate students of mathematics and related disciplines by exposing them to interdisciplinary problems and collaborations addressing questions of great technological importance. One of the drawbacks of the finite element and finite difference immersed boundary methods is that they are only first-order accurate due to the non-smoothness of the solution across the interface. Furthermore, very few mathematical analyses of these methods exist for time dependent problems and for fluid-structure interaction problems. The first part of the project involves the construction of higher-order FE-IBM algorithms and establishing a corresponding mathematical foundation to obtain rigorous time stability and a priori and a posteriori error estimates. The second part of the project deals with new finite element methods which are able to accurately capture solutions of elliptic interface problems with high-contrast coefficients in the case that the finite element mesh is not necessarily aligned with the interface. The goal here is to develop finite element methods with optimal convergence rates, where the constants hidden in these estimates are independent of the contrast and on how the mesh crosses the interface. Learn more about the grant.

Multiscale Methods in Beamed Energy Harnessing Applications

Air Force Office of Scientific Research (AFOSR)

Tilley, B., PI; Co-PI, Yakovlev, V.

2015-2018

We are interested in understanding how electromagnetic-radiation absorbing materials, either porous or designed with channels through which a coolant can flow, can be used to transfer the energy from electromagnetic radiation to a coolant. For the applications of interest, temperatures can reach 2000 K, and since the electrical conductivity of these materials depends on temperature, multiple steady temperatures are possible at the same input power. The research program centers on using asymptotic multi-scale methods including homogenization to formulate an effective-medium theory to describe the energy conservation and electric field amplitude propagation through this medium, for incompressible and compressible coolants. Results will be compared to GPU-accelerated finite-difference time-domain (FDTD) scripts in two spatial dimensions. Simulations in three dimensions will be done by implementing computational approaches from the 2D-FDTD schemes within a three-dimensional spatial framework in COMSOL Multi-physics.

Small Area Estimation at USDA's National Agricultural Statistics Service Research and Development Division

(RDD) at the National Agricultural Statistics Service (NASS)

Nandram, B.

2015-2016

This grant builds on Professor Nandram's prior experience working with NASS on Objective Yield Survey and Interview Yield Survey for forecasting US corn yields. As part of the new grant, Professor Nandram will participate in the new NASS small area estimation research program. In this program he will continue his work in applications of Bayesian small area estimation to government programs such as the National Health and Nutrition Examination Survey and the National Health Interview Survey to assess the health of the US population. The Research and Development Division (RDD) at the National Agricultural Statistics Service (NASS) employs a number of recent PhDs and researchers working on their dissertations. As part of the grant, Professor Nandram will also provide research mentoring for these individuals.

Topics in quasiconformal mappings and subelliptic PDE

National Science Foundation

Capogna, L.

2015-2018

SubRiemannian geometry and subelliptic partial differential equations (PDEs) are used to model real life systems where there is a constrained dynamics. Examples of such systems include the motion of robot arms, structural functions of the first layer of the mammalian visual cortex, the Black-Scholes model for financial markets and quantum computing. Geometric and analytic properties of such spaces are captured in a quantitative fashion by studying the behavior of certain families of transformations of the space into itself. This project aims at studying fine properties of such transformations. In particular, the proposed research will provide a theoretical basis for implementing numerical simulations of real-life system. Learn more about the grant.

Collaborative Research: Computational Models of Cilia and Flagella in a Brinkman Fluid

National Science Foundation

Olson, S.

2014-2017

Motile cilia and flagella are dynamic, elastic, biological structures that exhibit rhythmic motion. Through coordinated beating, cilia in the lung and respiratory tract help to clear the airway of potentially harmful particles and mucus. Impaired cilia motion can result in serious respiratory infection. This impairment can be caused by an altered fluid environment, a characteristic of cystic fibrosis, by defects in the cilia themselves as with primary ciliary dyskinesia, or by other respiratory diseases. Similarly, a sperm is only able to reach and fertilize an egg if its flagellum can propel it forward; impaired motility results in infertility. Cilia and sperm beating is highly dependent on the fluid environment, which contains fibrous, protein networks as well as chemical signals. Understanding this relationship between elastic structure and heterogeneous fluid is vital for development of delivery strategies for inhaled drugs and medicines, new contraceptives, and aiding in infertility due to reduced sperm motility. This project will focus on the development of new computational models and numerical methods that account for these fibrous, protein networks within the fluid to infer in vivo behavior of cilia and sperm. Recent experiments have shown that i) sperm-flagella waveforms are altered when immersed in fluids containing networks of large proteins and also with flagellar calcium concentration, ii) the previously-considered `watery' fluid surrounding airway cilia actually contains a network of large proteins, or, 'brush'. This project will focus on identifying emergent waveforms of sperm flagella and airway cilia when nonplanar bending and internal flagellar biochemistry are taken into account. Cilia and flagella will be modeled as slender, elastic, structures immersed in a fluid governed by the Brinkman equation. A regularized framework and fundamental solutions will be used to derive new numerical methods for various confined geometries. The new numerical methods will be computationally-efficient and developed for use on high-performance computing systems. This research will identify factors that modulate sperm progression towards the egg, explore various waveforms of cilia and sperm, and investigate how a 'brush' in the fluid surrounding cilia might affect transport of particles within that fluid. The award will also fund RA’ships for two years and provide undergraduate student support. This is part of a 2 PI collaborative proposal, which also include a separate award to Dr. Karin Leiderman at UC Merced.

Learn more about the grant