WPI Mathematics Institute for Secondary Teaching (MIST)

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LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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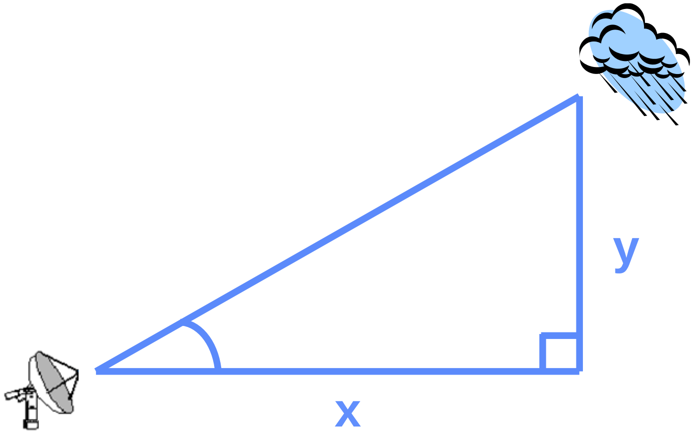
Outline

- Introduction
- Radar: Calculating Distances
- Radar: Calculating Precipitation Intensity
- Calculating Storm Echo Top Heights
- Polar and Cartesian Coordinates
- Mapping Projections

Introduction

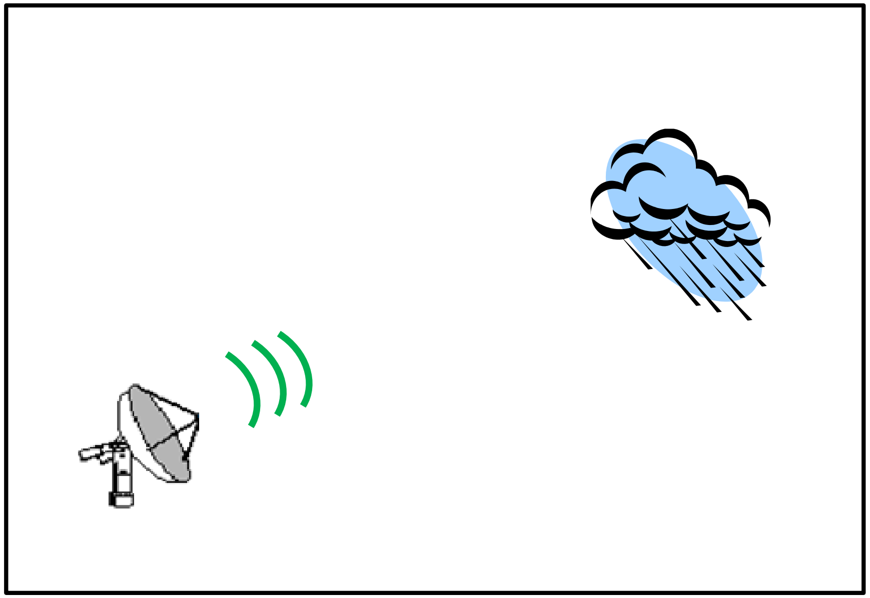
- **High school math is used for and provides the foundation of work at Lincoln**
- **Lots of high school math applications to weather radar and forecast generation**
- **If we didn't have a good grasp of trigonometry, geometry, algebra, and calculus, we couldn't do our jobs!**



Radar Basics

Weather radars are used to detect, locate, and measure intensity of precipitation

WSR-88D
Weather Radar



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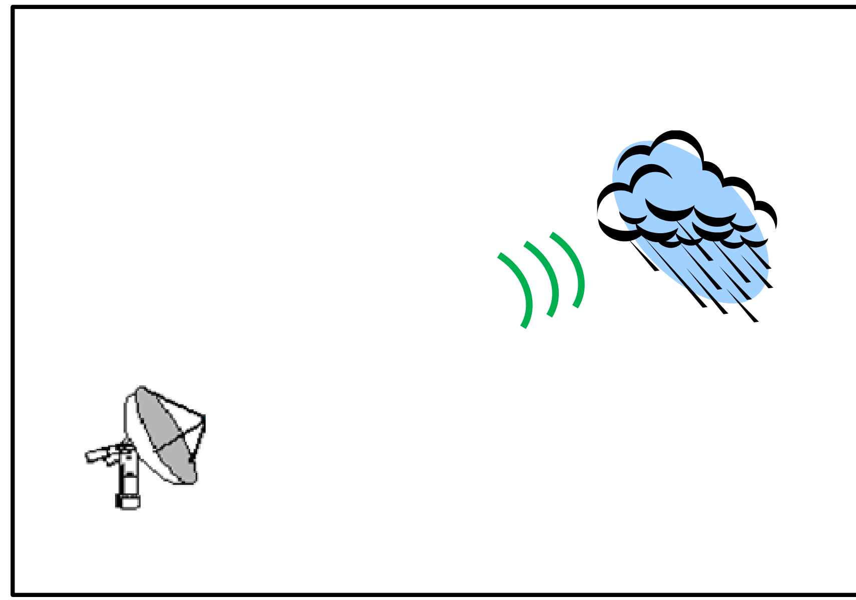

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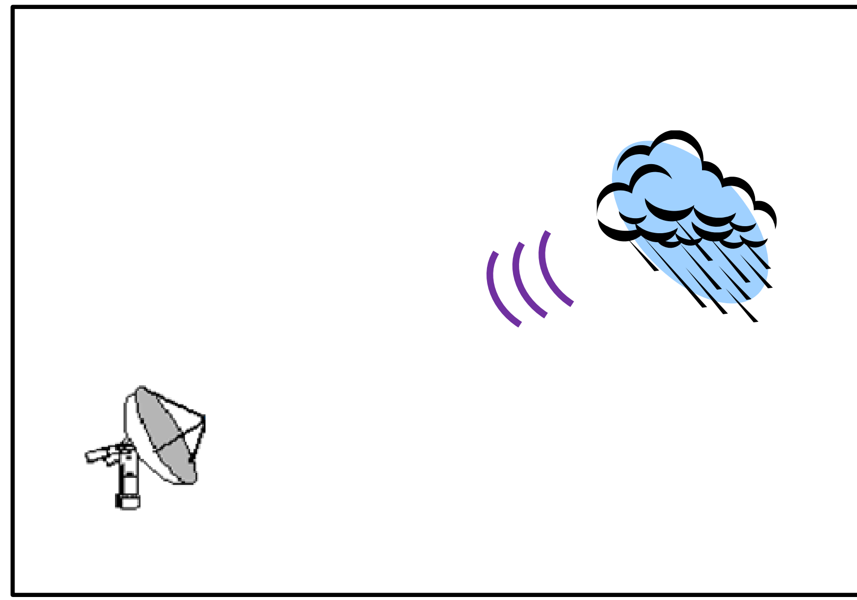

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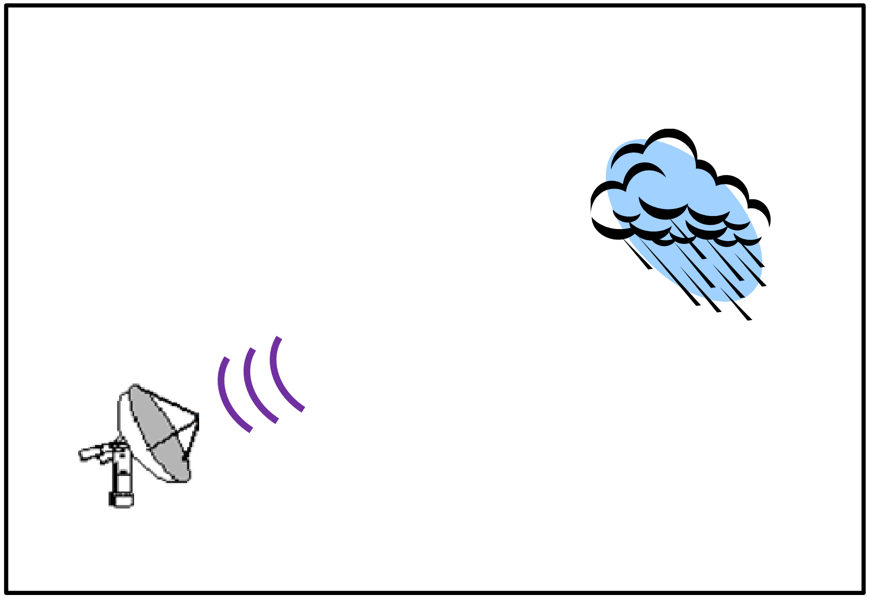
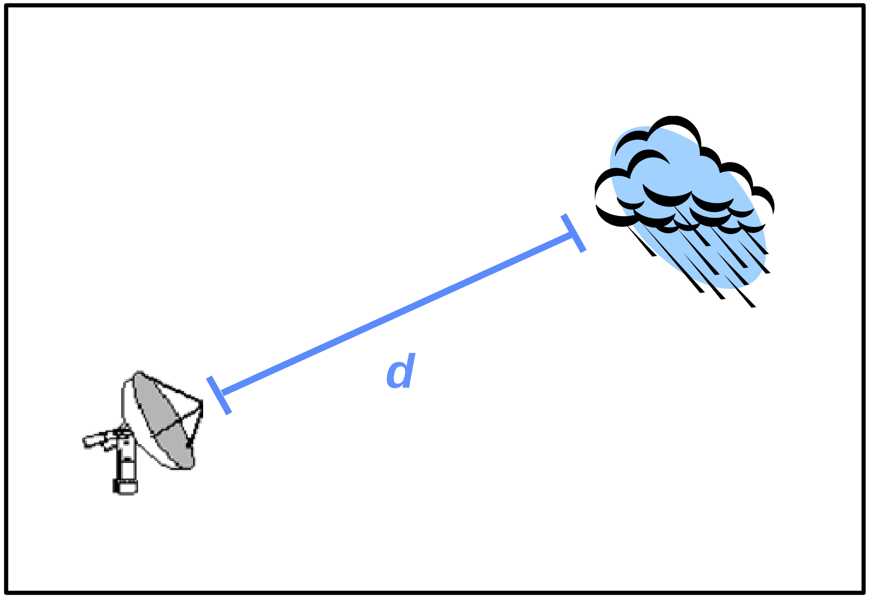


Calculating Distances



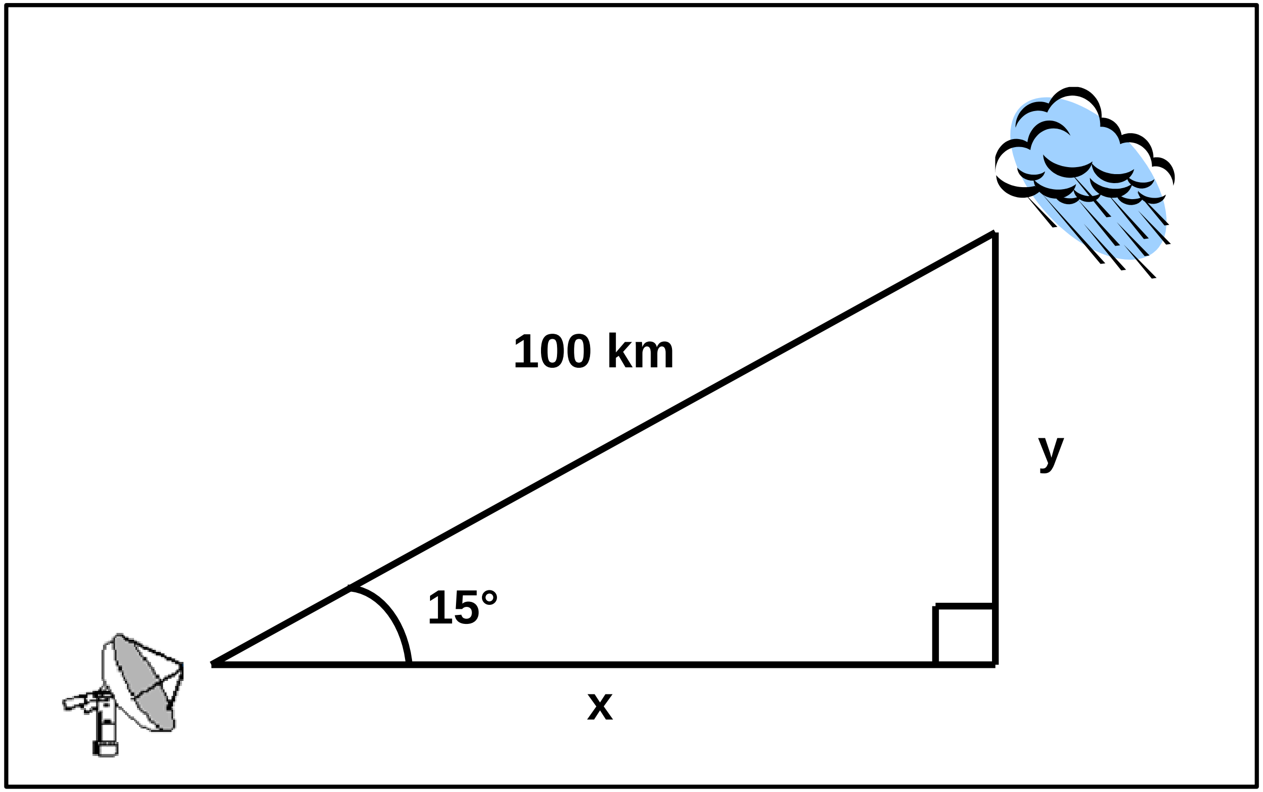
distance = rate * time

$$d = c * \frac{t}{2}$$

***d*: distance**
c*: speed of light *through air
***t*: round-trip time for pulse to hit target and return**

If you can measure elapsed time accurately, you can calculate distance accurately!

Calculating Distances (cont.)

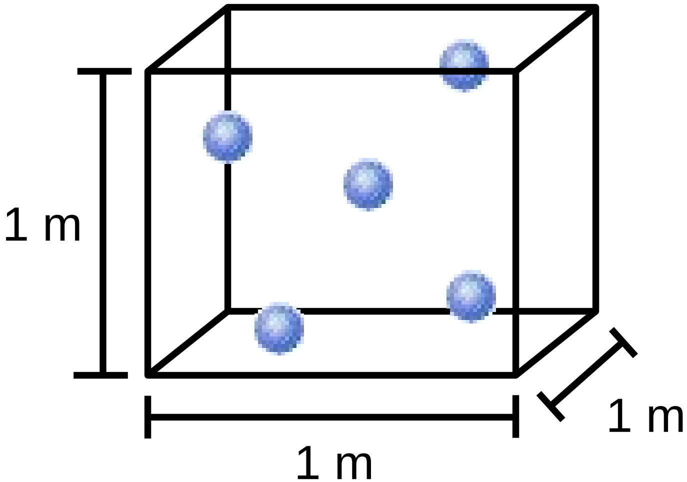


$$\sin 15° = \frac{y}{100}$$

$$\cos 15° = \frac{x}{100}$$

Note! In reality, radar beams usually bend slightly towards Earth, and the Earth is not flat.

Calculating Precipitation Intensity



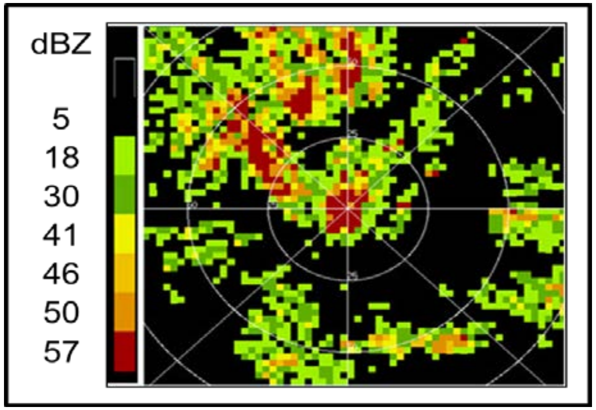
Water droplet with diameter *d* mm

$$Z \propto \sum_i d_i^6$$

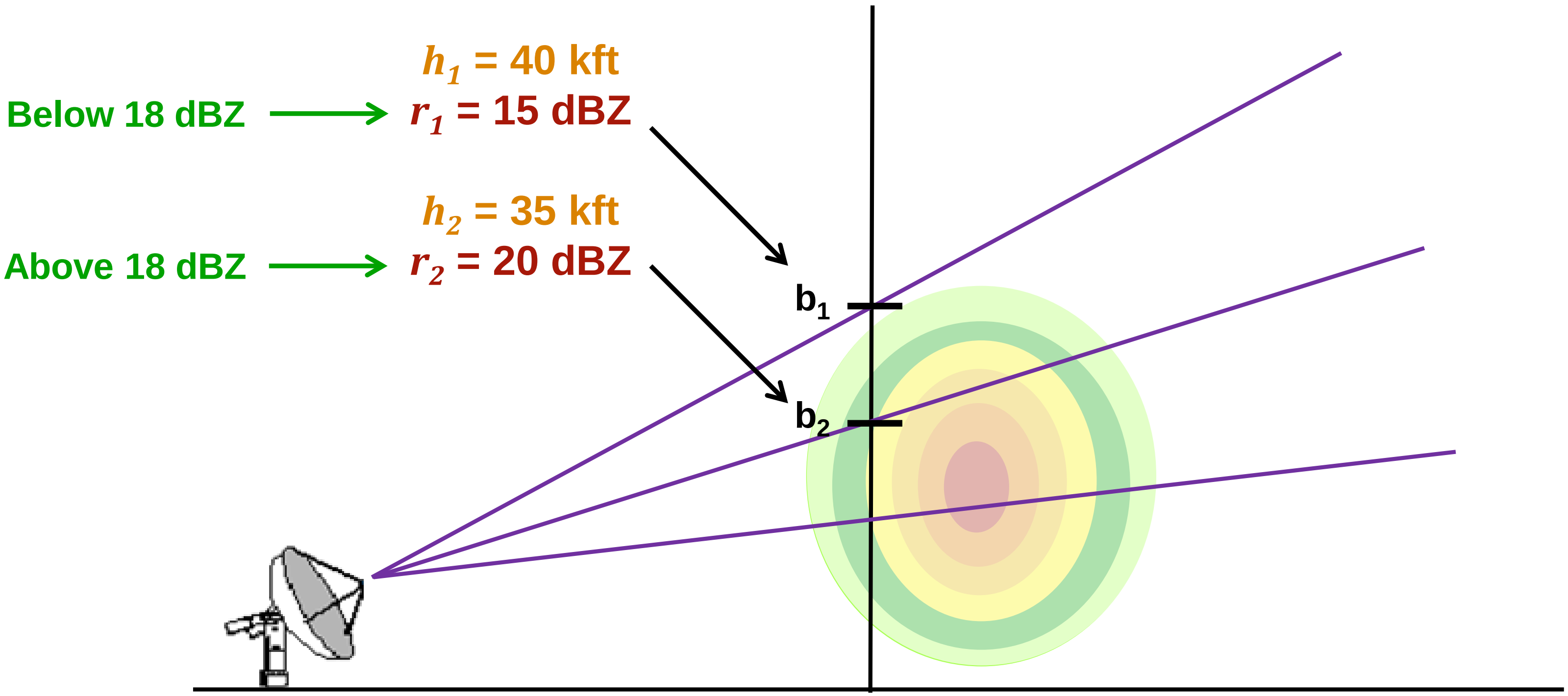
Z: Units of $\mathrm{mm^6 m^{-3}}$

$$dBZ = 10 \log_{10} \frac{Z}{1\ \mathrm{mm^6 m^{-3}}}$$

dBZ: dimensionless



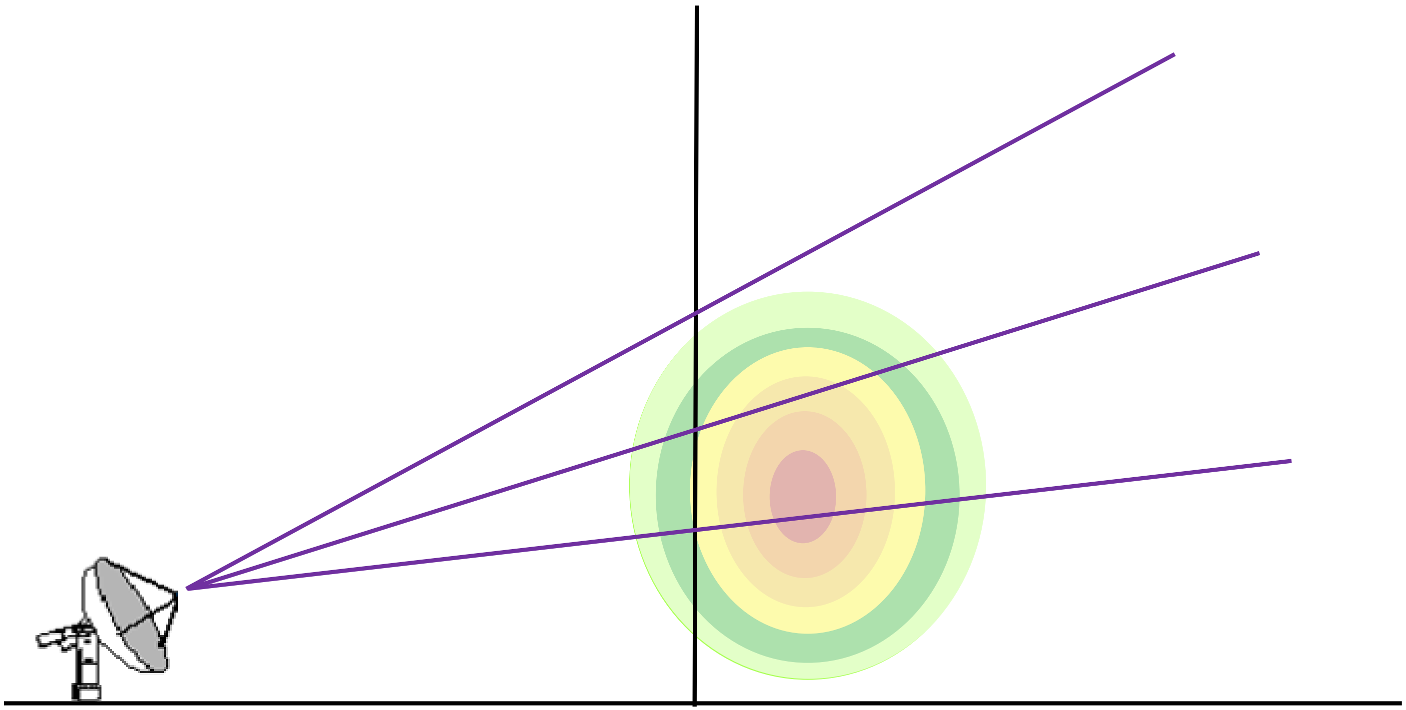
Calculating Storm Echo Top Heights



$$e = h_2 + \frac{18 - r_2}{r_1 - r_2}(h_1 - h_2)$$

$$= 37 \text{ kft}$$

Calculating Storm Echo Top Heights



Similar calculations can be carried out for different ranges and azimuths!

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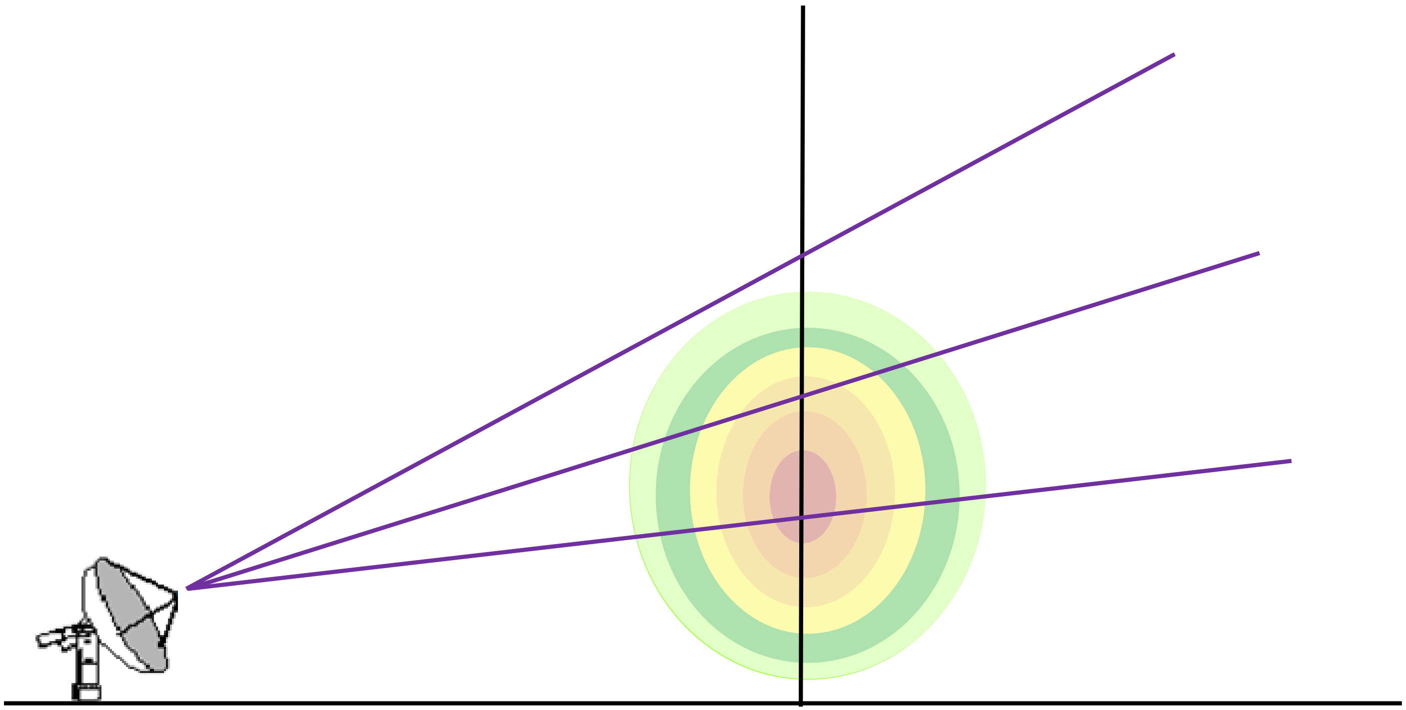

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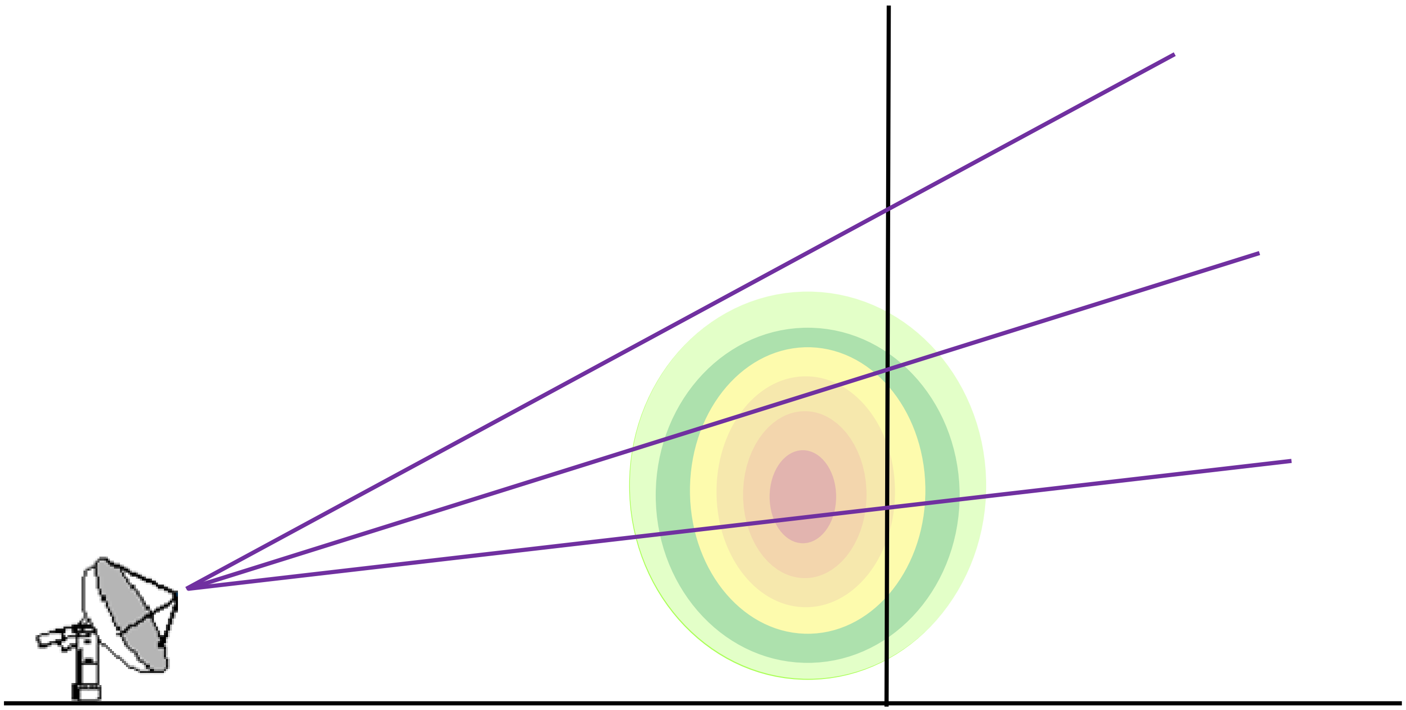

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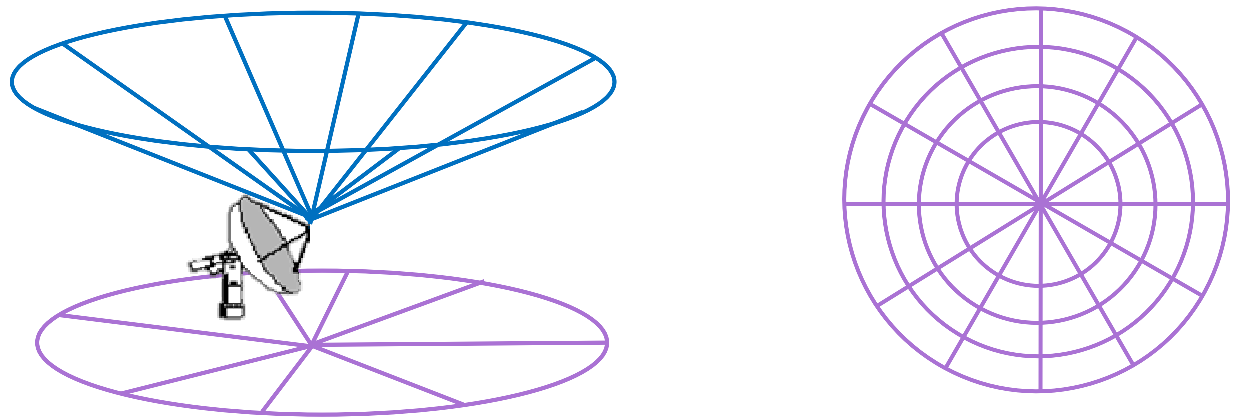
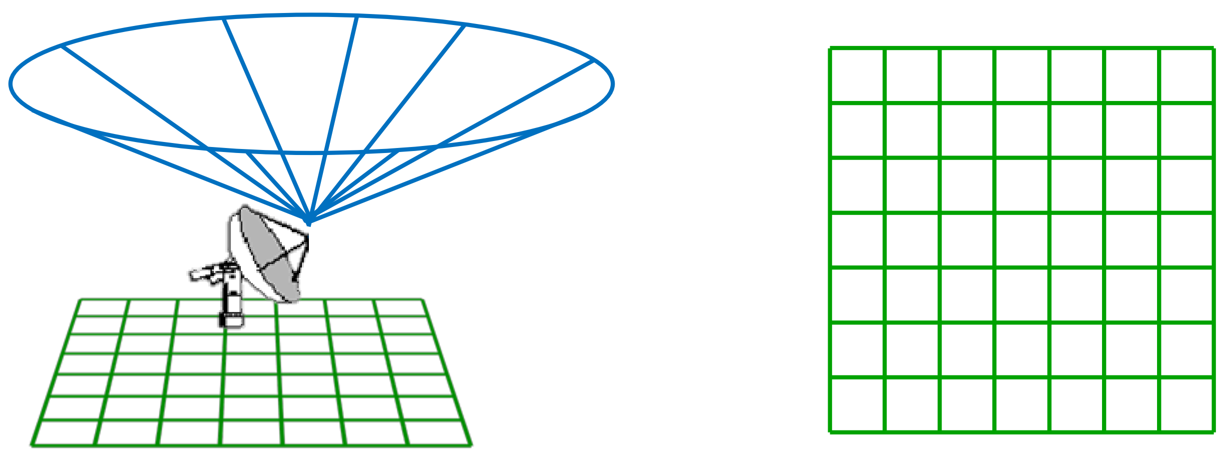
Polar and Cartesian Coordinates

Polar Coordinates



Cartesian Coordinates



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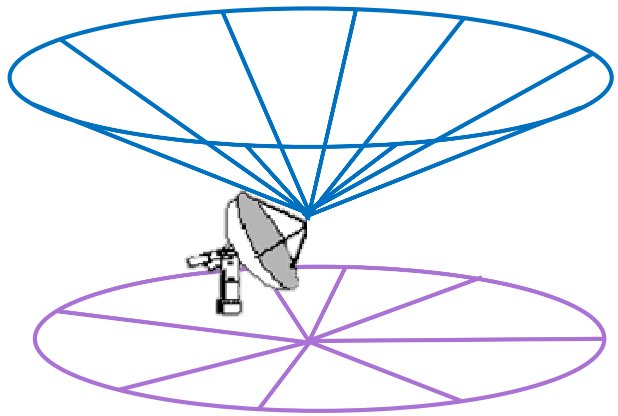

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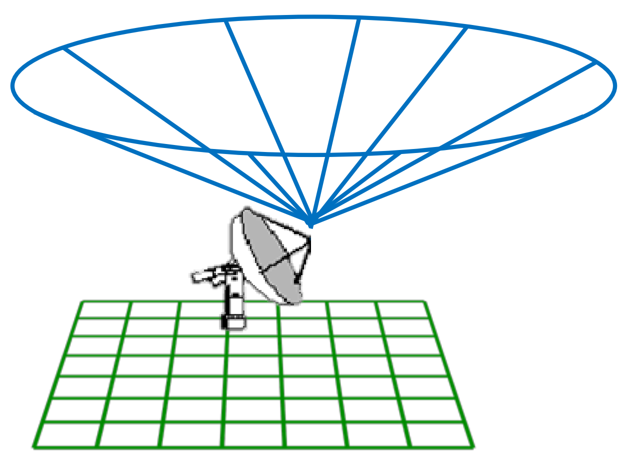
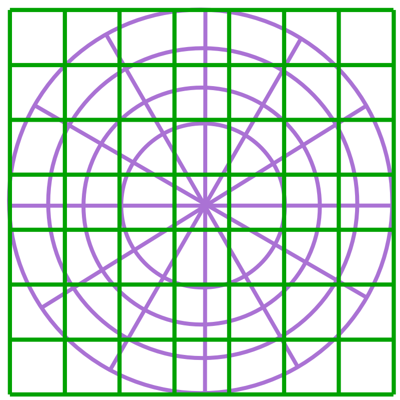
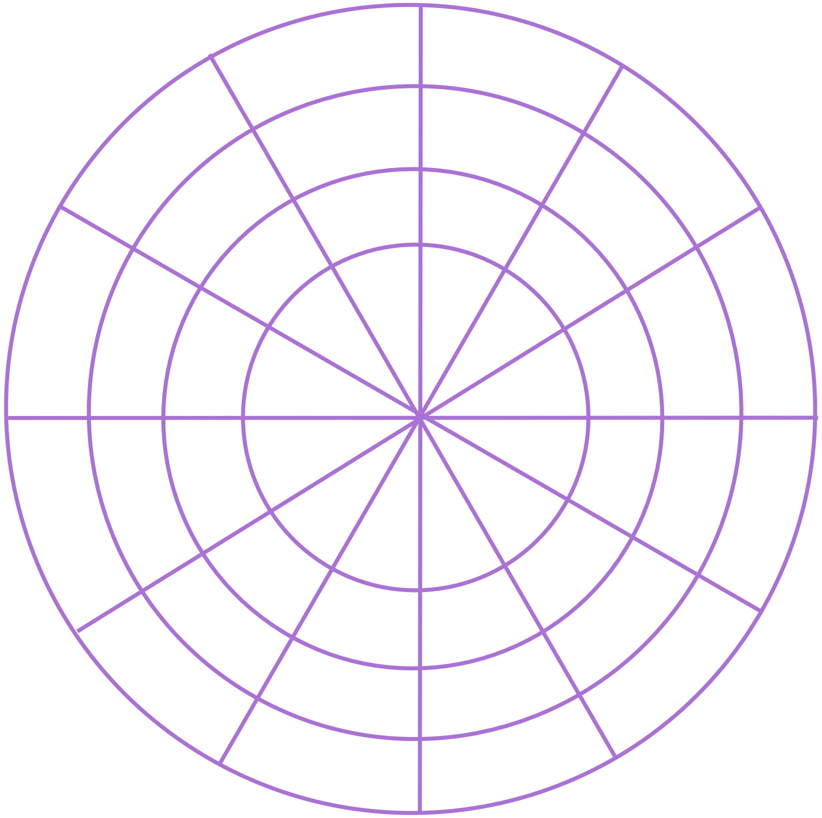




Converting from Polar to Cartesian



How do we go from a polar coordinate to a point on a Cartesian grid?

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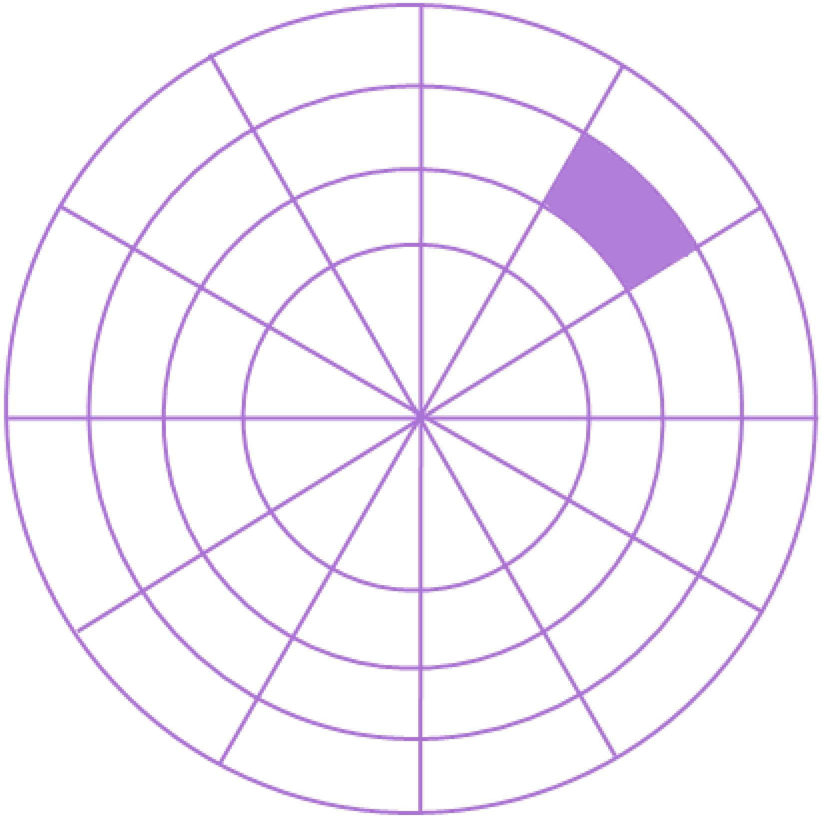

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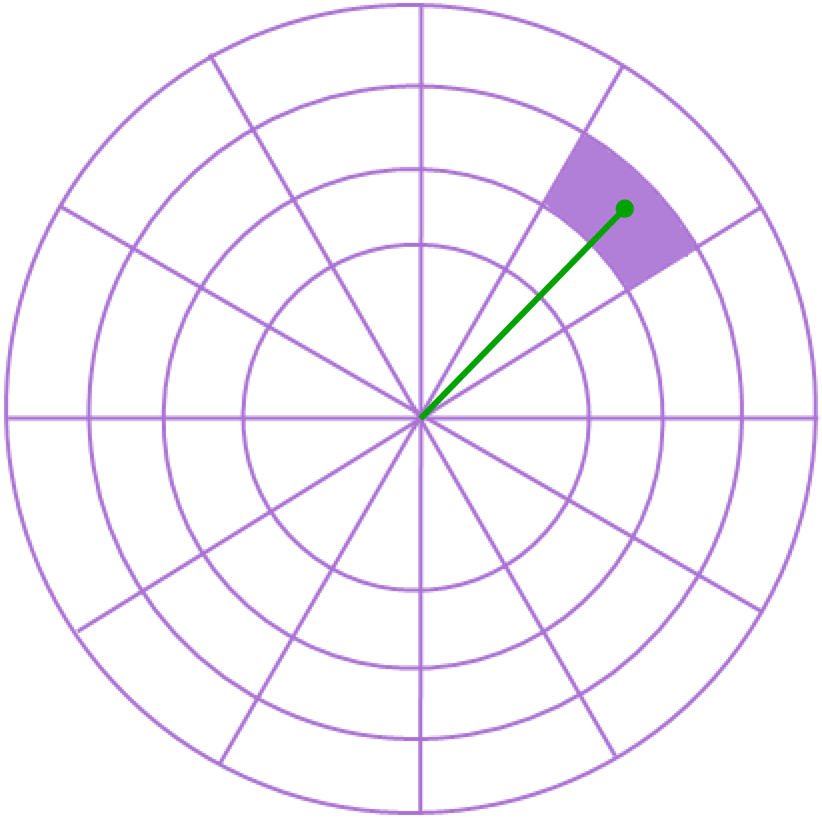

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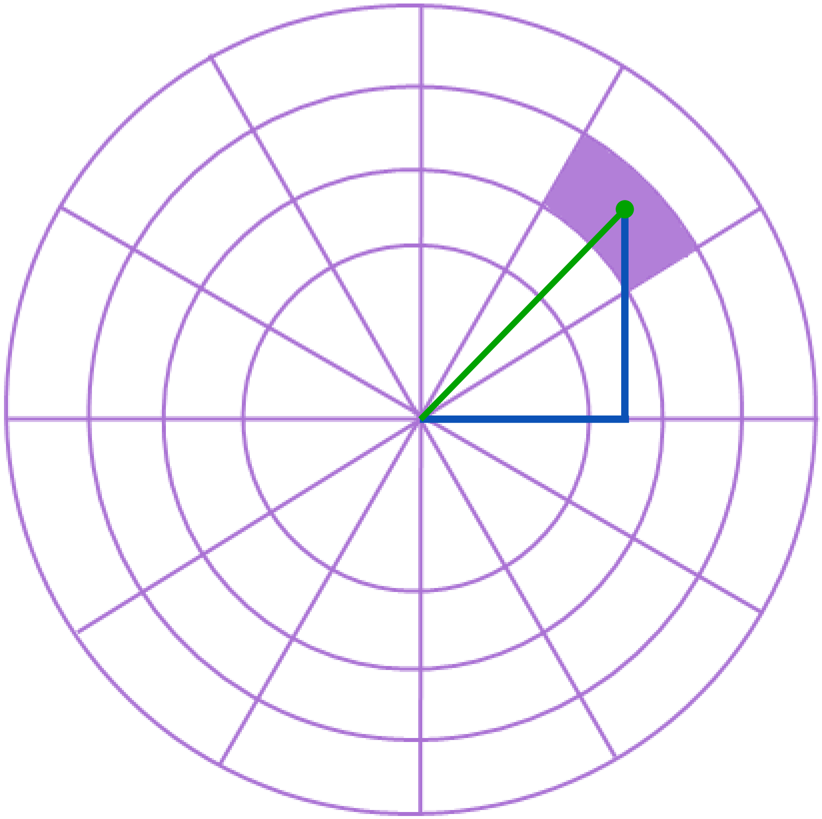
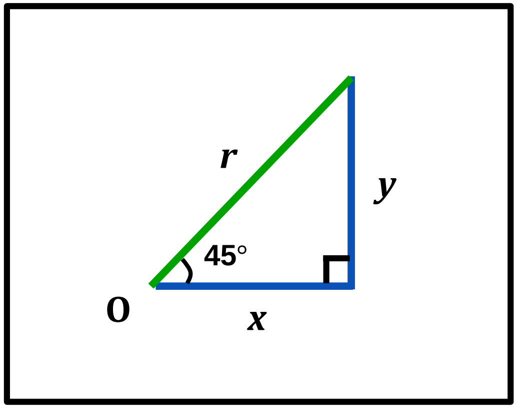




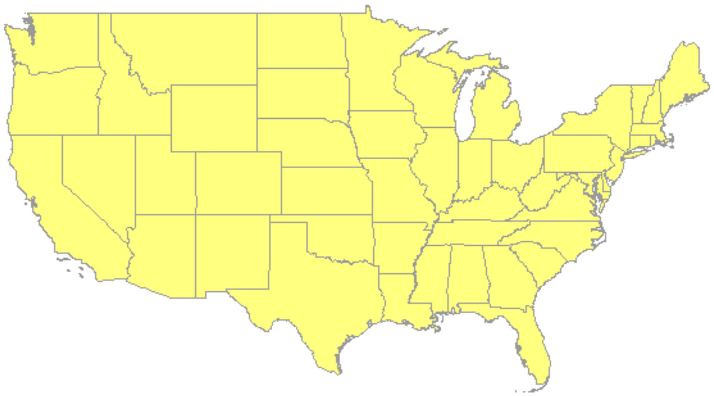
$$\sin 45^{\circ} = \frac{y}{r}$$

$$\cos 45^{\circ} = \frac{x}{r}$$

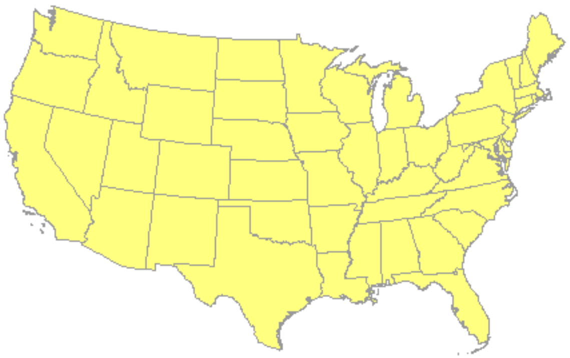
Mapping Projections

Different organizations can use different mapping projections.

How do we compare forecasts on maps that don't look the same?



Mercator Projection



Stereographic Projection

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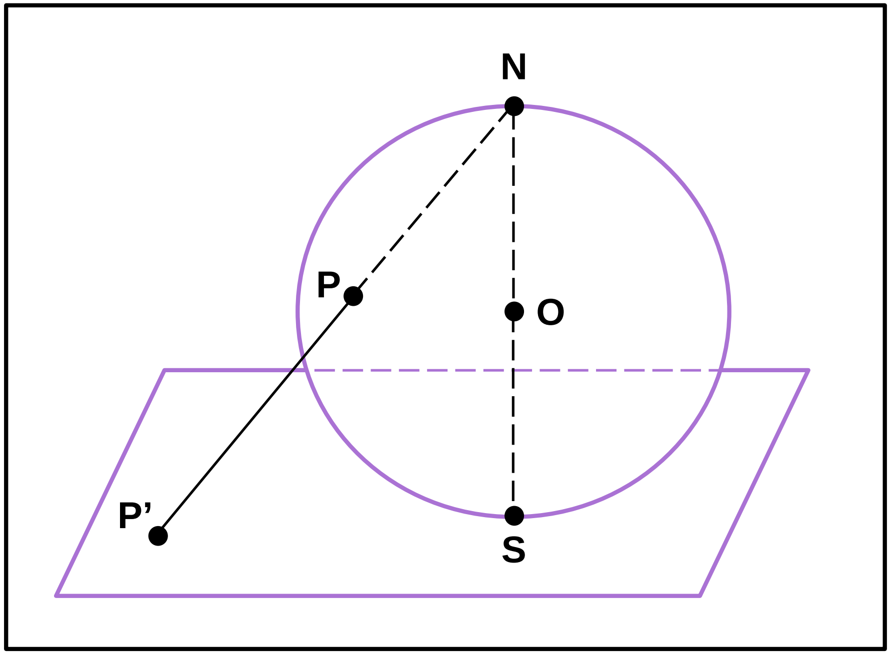
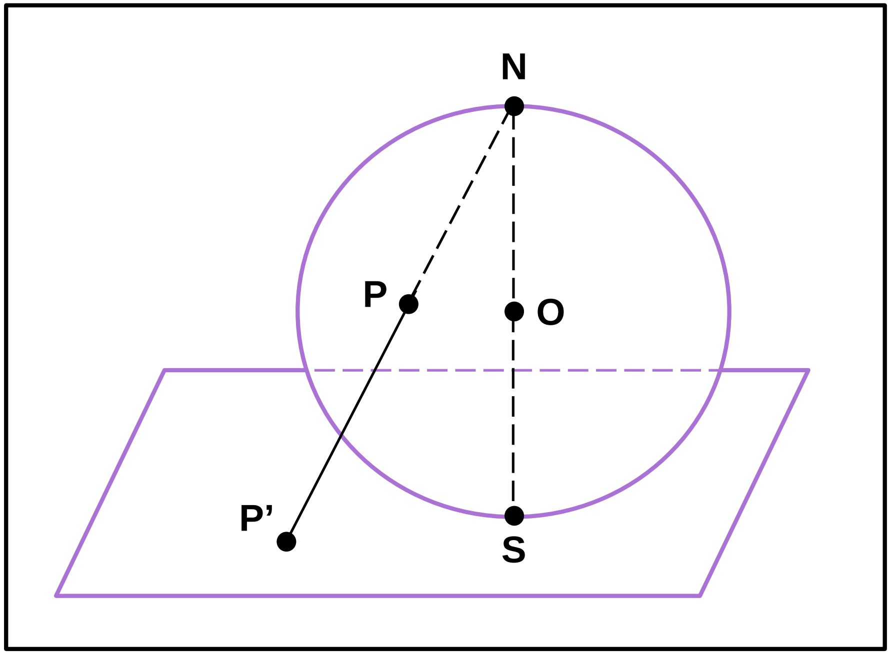


A point P on the sphere is mapped to a unique point P’ on the plane

That is, a point P on the Earth is mapped to a unique point P’ on the map

Stereographic Projections



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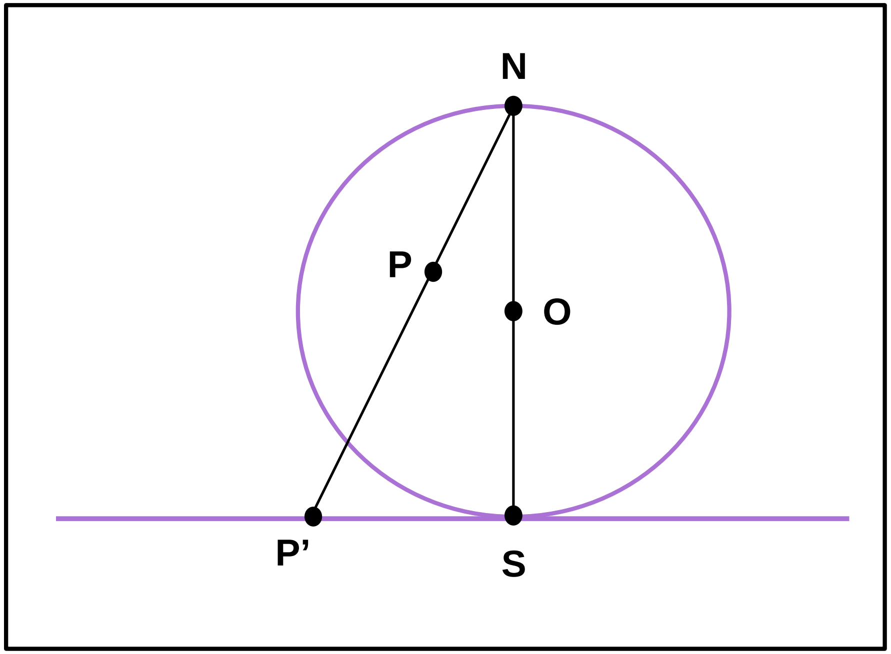
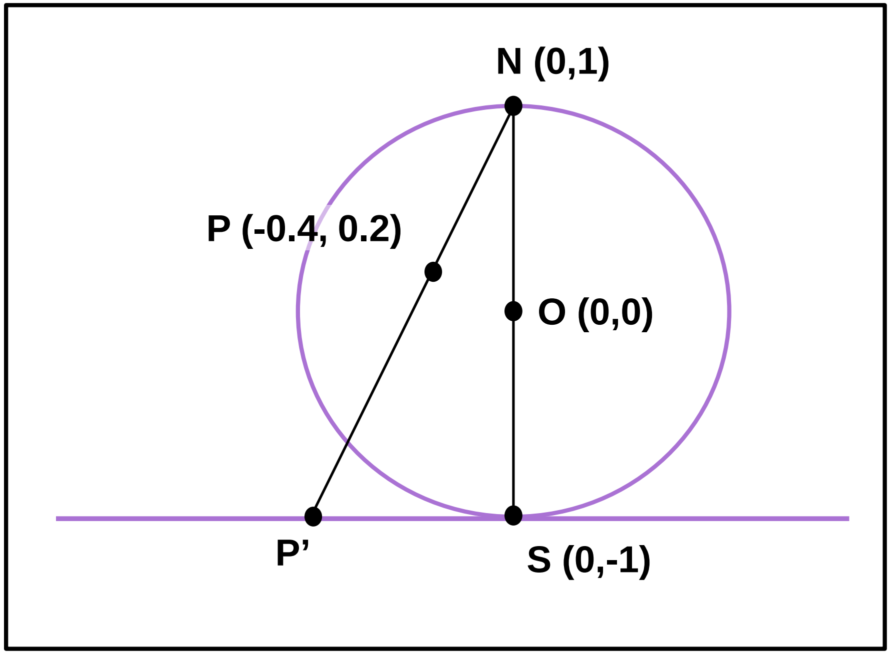
Stereographic Projections: Simplified



A point P on the circle is mapped to a unique point P' on the line

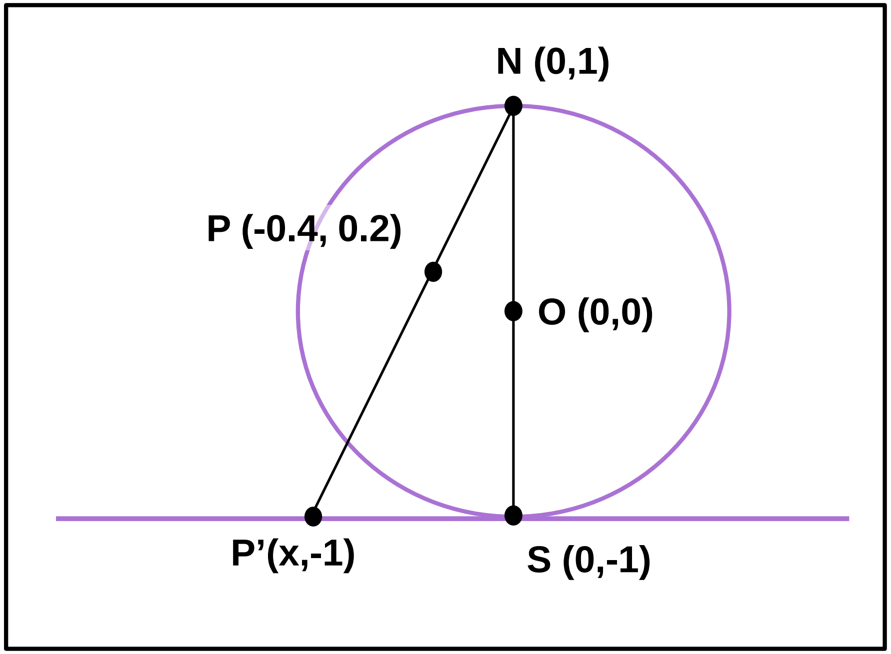
Stereographic Projections: Simplified



Two ways to find P':

1) Find equation of line from N to P
2) Use similar triangles

Finding Line from N To P



$$y = mx + b$$

Plug in values for N and P:

$$1 = m(0) + b$$
$$0.2 = m(-0.4) + b$$

Solve for *m* and *b:*

$$m = 2, b = 1$$

Write equation for the line:

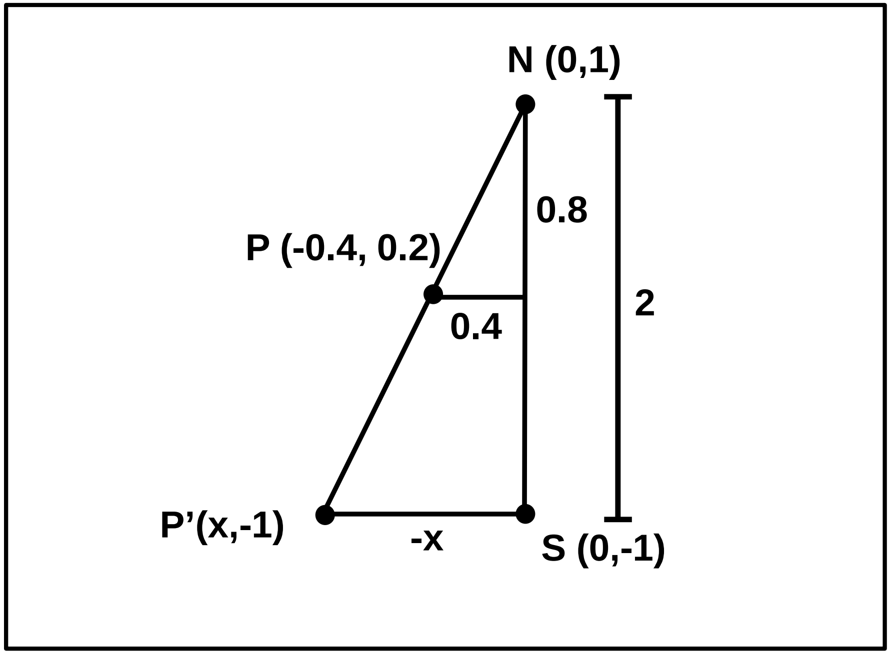
$$y = 2x + 1$$

Find x-coordinate of P’:

$$-1 = 2(x) + 1$$
$$x = -1$$

P’: (-1, -1)

Using Similar Triangles



Using similar triangles:

$$\frac{2}{-x} = \frac{0.8}{0.4}$$

Cross multiply:

$$2 * 0.4 = 0.8 * (-x)$$

$$0.8 = -0.8x$$
$$x = -1$$

P': (-1, -1)

You could imagine extending these ideas to add another dimension!

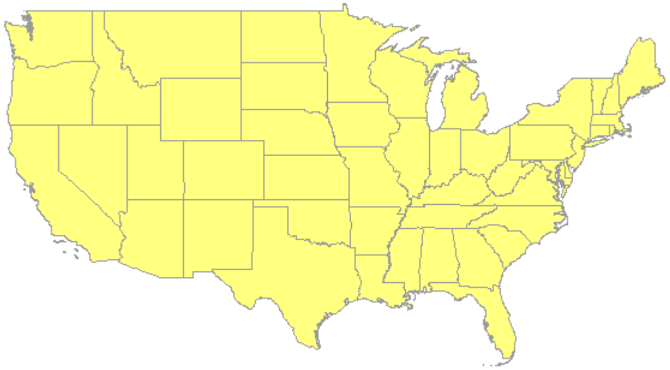
Converting Between Projections

If φ is latitude, λ is longitude:

Mercator projection:

$$x = \lambda$$

$$y = \frac{1}{2} \ln\left(\frac{1 + \sin\varphi}{1 - \sin\varphi}\right)$$

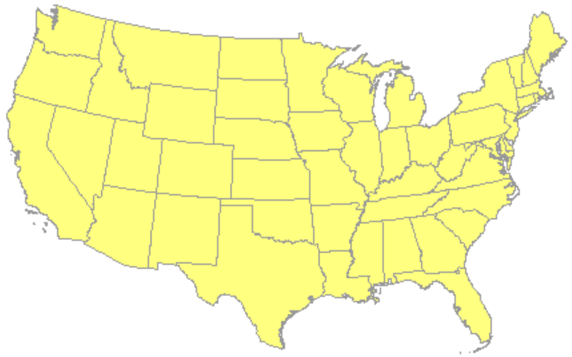


If φ is latitude, λ is longitude, φ_1 is central latitude, λ_0 is central longitude, and *R* is local radius of Earth:

Stereographic projection:

$$x = \frac{2\,R \cos\varphi \sin(\lambda - \lambda_0)}{1 + \sin\varphi_1 \sin\varphi + \cos\varphi_1 \cos\varphi \cos(\lambda - \lambda_0)}$$

$$y = \frac{2\,R\,[\cos\varphi_1 \sin\varphi - \sin\varphi_1 \cos\varphi \cos(\lambda - \lambda_0)]}{1 + \sin\varphi_1 \sin\varphi + \cos\varphi_1 \cos\varphi \cos(\lambda - \lambda_0)}$$



Summary

- **High school math has many applications to weather radar and forecast generation**

- **Calculating storm position/intensity and disseminating that information would not be possible without high school math**

- **Without math, we'd be left sticking our heads out the window for weather information!**