Outline

• Introduction

• Radar: Calculating Distances

• Radar: Calculating Precipitation Intensity

• Calculating Storm Echo Top Heights

• Polar and Cartesian Coordinates

• Mapping Projections
Introduction

• High school math is used for and provides the foundation of work at Lincoln

• Lots of high school math applications to weather radar and forecast generation

• If we didn’t have a good grasp of trigonometry, geometry, algebra, and calculus, we couldn’t do our jobs!
Weather radars are used to detect, locate, and measure intensity of precipitation.
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WSR-88D
Weather Radar
Calculating Distances

distance = rate * time

\[ d = c \frac{t}{2} \]

- **d**: distance
- **c**: speed of light *through air*
- **t**: round-trip time for pulse to hit target and return

If you can measure elapsed time accurately, you can calculate distance accurately!
Calculating Distances (cont.)

\[ s_1 = y \]
\[ c_1 = x \]

100 km

15°

\[ \sin 15° = \frac{y}{100} \]
\[ \cos 15° = \frac{x}{100} \]

Note! In reality, radar beams usually bend slightly towards Earth, and the Earth is not flat.
Calculating Precipitation Intensity

Water droplet with diameter $d$ mm

$$Z \propto \sum_i d_i^6$$

$Z$: Units of mm$^6$m$^{-3}$

$$dBZ = 10 \log_{10} \left( \frac{Z}{1 \text{ mm}^6 \text{m}^{-3}} \right)$$

$dBZ$: dimensionless
Calculating Storm Echo Top Heights

\[ e = h_2 + \frac{18 - r_2}{r_1 - r_2} (h_1 - h_2) \]

\[ = 37 \text{ kft} \]

Below 18 dBZ
- \( h_1 = 40 \text{ kft} \)
- \( r_1 = 15 \text{ dBZ} \)

Above 18 dBZ
- \( h_2 = 35 \text{ kft} \)
- \( r_2 = 20 \text{ dBZ} \)
Calculating Storm Echo Top Heights

Similar calculations can be carried out for different ranges and azimuths!
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Polar and Cartesian Coordinates

Polar Coordinates

Cartesian Coordinates
Polar and Cartesian Coordinates

Polar Coordinates

Cartesian Coordinates
Converting from Polar to Cartesian

How do we go from a polar coordinate to a point on a Cartesian grid?
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\[
\sin 45^\circ = \frac{y}{r} \\
\cos 45^\circ = \frac{x}{r}
\]
Different organizations can use different mapping projections.

How do we compare forecasts on maps that don’t look the same?

Mercator Projection

Stereographic Projection
Stereographic Projections

A point $P$ on the sphere is mapped to a unique point $P'$ on the plane.

That is, a point $P$ on the Earth is mapped to a unique point $P'$ on the map.
Stereographic Projections

A point $P$ on the sphere is mapped to a unique point $P'$ on the plane.

That is, a point $P$ on the Earth is mapped to a unique point $P'$ on the map.
A point P on the circle is mapped to a unique point P' on the line.
Two ways to find $P'$:

1) Find equation of line from $N$ to $P$

2) Use similar triangles
Finding Line from N To P

\[ y = mx + b \]

Plug in values for N and P:
\[ 1 = m(0) + b \]
\[ 0.2 = m(-0.4) + b \]

Solve for \( m \) and \( b \):
\[ m = 2, b = 1 \]

Write equation for the line:
\[ y = 2x + 1 \]

Find \( x \)-coordinate of \( P' \):
\[ -1 = 2(x) + 1 \]
\[ x = -1 \]

\( P' \): (-1, -1)
Using similar triangles:
\[
\frac{2}{-x} = \frac{0.8}{0.4}
\]

Cross multiply:
\[
2 \cdot 0.4 = 0.8 \cdot (-x)
\]

\[
0.8 = -0.8x
\]

\[
x = -1
\]

\[
P': (-1, -1)
\]

You could imagine extending these ideas to add another dimension!
Converting Between Projections

If $\varphi$ is latitude, $\lambda$ is longitude:

**Mercator projection:**

\[
x = \lambda
\]
\[
y = \frac{1}{2} \ln \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right)
\]

If $\varphi$ is latitude, $\lambda$ is longitude, $\varphi_1$ is central latitude, $\lambda_0$ is central longitude, and $R$ is local radius of Earth:

**Stereographic projection:**

\[
x = \frac{2 R \cos \varphi \sin(\lambda - \lambda_0)}{1 + \sin \varphi_1 \sin \varphi + \cos \varphi_1 \cos \varphi \cos(\lambda - \lambda_0)}
\]
\[
y = \frac{2 R [\cos \varphi_1 \sin \varphi - \sin \varphi_1 \cos \varphi \cos(\lambda - \lambda_0)]}{1 + \sin \varphi_1 \sin \varphi + \cos \varphi_1 \cos \varphi \cos(\lambda - \lambda_0)}
\]
High school math has many applications to weather radar and forecast generation.

Calculating storm position/intensity and disseminating that information would not be possible without high school math.

Without math, we’d be left sticking our heads out the window for weather information!