Title/Essential Question: Can you afford to buy your dream car?

Authors:
Barbara Chyen, Carol Cohen, Sherrie-Lynne Belanger, Cara Lake, Jennifer Gould

Project overview
You are just about to get your first brand new car. You have so many ideas about what you’d like to get. Can you afford your choice of car with the job you have? You will use the Present Value formula to calculate the number of months it will take you to pay off the car loan based on the amount of the loan, the amount of the monthly car payment and an interest rate. What’s the actual amount of the interest that you will be paying the bank?

Background resources
- [http://www1.salary.com/Entry-Level-salaries.html](http://www1.salary.com/Entry-Level-salaries.html)

Mathematics involved
- Solving logarithmic equations algebraically
  - Convert exponential equation to log equation to isolate variable
  - Or, use log properties to isolate variable (power property/change of base rule)
- Solving logarithmic equations graphically (setting correct window, use calculator tools to find the point of intersection of two graphs)
- Percentages
- Vocabulary (present value, interest, budget, etc)

Secondary areas possibly related
- Personal Finance (banking, budgeting, Present Value)
- Economics
- Presentation skills
- Pre-Calculus/Algebra II

Common Core - Standards for Mathematical Content
- **F.BF.5 (+)** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents
- **F.LE.4** For exponential models, express as a logarithm the solution to \(ab^ct = d\) where \(a, c,\) and \(d\) are numbers and the base \(b\) is 2, 10, or \(e\); evaluate the logarithm using technology.
- **F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

**Standards for Mathematical Practice that will be used in this project:**
- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning?

**Physical or electronic props needed**
- Calculator
- Computer with internet access
- Slips of paper with different jobs with salaries (pick out of a hat)

**Expectations of students**
- Be able to work/have productive discussions with others effectively
- Think critically
- Solve equations using logarithms to find # of monthly payments
- Understand the relationship between exponential functions and logarithmic functions
- Convert between units
- Find the total amount of interest paid to the bank (scary!)

**What is the Plan?**
- Start with an individual brainstorm or think/pair/share after posing the essential question: You want to buy your dream car. Can you afford it?
  - What questions will you need answered in order to answer the essential question of this project?
  - What information will I need?
  - How will I get this information?
  - Follow up with a full class discussion listing needs
- Provide project description with outlined steps something like this...
  - Students will be randomly assigned (picking out of a hat) a career and entry level salary such as... teacher, admin assistant, laborer, engineer, fitness trainer, cosmetologist/barber, etc
  - What is an appropriate monthly budget for your car payment, given your salary (discussion hopefully happened earlier that it is roughly 10%) - from here calculate monthly car payment (could include insurance if you want as an extension).
  - Student shops for their ideal car including bells and whistles. What is the cost?
Either give students interest rate and loan term or as an extension students could research on their own.

Provide PV formula and define variables

Use the following formula:  
\[ PV = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

- \( PV \): present value of the loan or the amount of the loan (cost of car)
- \( PMT \): monthly payment
- \( n \): number of payments (this is what you are finding)
- \( i \): interest rate as a decimal
  
\[ i = \frac{\text{interest rate as a decimal}}{\text{number of times compounded per year}} \]

(Since you will be making monthly payments, the number of times compounded per year = 12.)

Students use formula to calculate the number of payments (n) it will take before they can pay off their car loan. This should be solved algebraically using logarithms. Students will also solve graphically to practice calculator skills.

Convert loan term to months and compare it to n. Can you afford it?

Have students interpret results. What does my solution mean?

Calculate the total interest you will pay the bank. (Ouch!)

Present results in a report or presentation

Other thoughts and options
- Add in car insurance to monthly payment?
- Work as an independent project at home or group/pair in class?